

WHY MATHS CLUB ?

By

P. K. SRINIVASAN

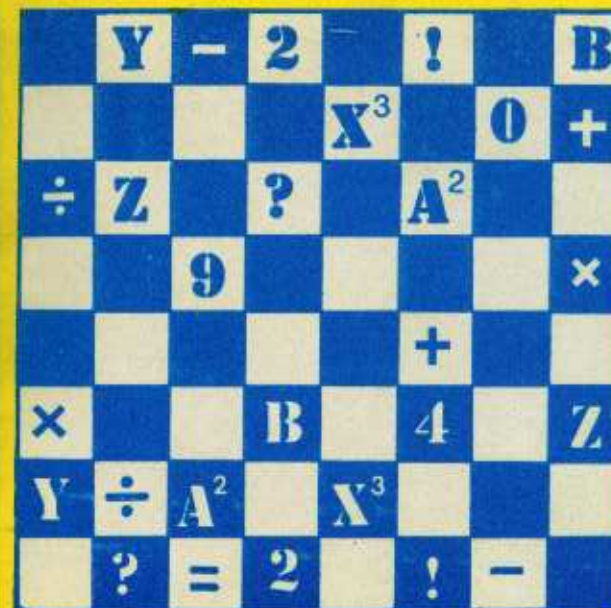
A
HAND BOOK OF GUIDELINES
FOR
TEACHERS IN MATHEMATICS, HEADS
INSPECTORS AND ADMINISTRATORS.

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P.K. SRINIVASAN

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PREFACE

MATHEMATICS continues to widen its frontiers of abstraction and application at an astoundingly fast rate with its geometries, algebras and arithmetics and their inter disciplines. This is the golden age of mathematics which has witnessed the advent of the computer representing as it does in all its glory and grace the narrowing gap between abstraction and application. The increasing employment of mathematical tools in hitherto undreamt of fields of human endeavour like biology, economics, sociology, business management, etc., has dramatised its importance in school curriculum.

Progress in science is reflected in terms of new devices and gadgets coming into the market and so it becomes familiar to one and all, whereas in mathematics progress is in terms of conceptual clarity, greater abstraction and newer application which mostly the university men of mathematics can alone be expected to know. Neither do the school teacher, by and large, get the urge or feel the pressure to know these tremendous developments in mathematical thought nor are they given opportunities to get exposed to them. A few interested teachers, however, maintain through private study and reflection their zeal for learning new things, teaching in new ways and transmitting new trends for the sheer joy of seeing children enjoy themselves through intellectual adventures. The influence of such teachers needs to be allowed to spread by means of a built in mechanism in the functioning of schools and colleges. It requires extraordinary guts to work in an unfavourable climate and so it should not be considered a luxury to give devoted and enterprising teachers the necessary administrative support.

There is a stir and ferment in the atmosphere of learning mathematics in schools all over the world and it needs to be channelled into constructive and creative endeavours so that the mathematically gifted children could be detected and taken care of early, with a view to conserving the most valuable human resources a country has in the realm

of mathematics for the benefit of all. Talents can then be seen to surface for participation in International Olympiads which encourage and foster mathematical gifts in youngsters and in which not many countries participate today for lack of good standards and high attainments of insightful learning.

It is an acknowledged fact that the gifted children at all levels of school education seldom get gifted teachers to guide them and do not find the fare provided in the mathematics syllabus and presented in the classroom challenging enough to draw forth the best in them. Of course the syllabus cannot be enriched on that account as it would then, militate against the interests of the average and the slow learners. This problem can be solved only through school or college mathematics clubs sub-junior, junior, senior and super-senior catering respectively to the needs of primary, middle, high and higher secondary school pupils.

As one who has successfully pioneered in organising maths clubs, expositions and fairs since 1949 and in the light of the tremendous enthusiasm roused in pupils as witnessed during my service in India, USA and Nigeria, I would like to make an appeal to the Central and State Departments and Ministries of Education not to rest content with encouraging science clubs alone in schools but to direct incorporation of running maths clubs in the annual programme of work of every school and teacher training institution. The plea or excuse that science includes mathematics is erroneous as mathematics is often bypassed for want of time, interest, etc. Moreover, mathematical and scientific avenues to truth are different and they need separate and special nurturing, though interdisciplinary programmes could and should be arranged. Further, much more work, creative right from the primary level onwards, can be done in a mathematics club with much less expenditure than in a science club. Though science is spectacular, nothing serious and deep could be achieved without emphasizing its mathematical basis in these days of quantification and hence control.

Mathematical gifts deserve to be fostered and applauded like the gifts in arts and music, athletics and dance. Time

has come when mathematics has to be taught not as an exam passing subject alone but as an enjoyable subject. Teachers who have unfortunately not had this approach in their school and college days should consider it their duty to refashion themselves in such a way as not only to cover the portions but also to uncover the beauties in them. Mere exhortations and pious appeals from higher authorities do not work. The first step is to set the ball rolling by getting a mathematics club started in every school. The Inspection Manual needs to be revised to make it obligatory on the part of the inspecting officers to call for the materials put up in the annual exhibitions held, periodical bulletins issued and minutes of meetings conducted and to contact the participants and ascertain their achievements, reactions and comments for recording in their inspection reports. Parent-teacher association meetings in schools may start with mini-math exposition.

Mathematics clubs have another important role to play by creating a favourable climate for introducing changes in the mathematics curriculum and bringing to the notice of school pupils, contributions of gifted youngsters in other parts of the world and new discoveries of mathematicians within the levels of their understanding appearing in periodicals and enrichment books.

The aim of this handbook is to outline clearly the ways and means of organising and running mathematics clubs in schools and colleges under all circumstances in developing countries. Each unit on a club activity carries a sample programme to serve as a guideline, supplemented by a list of books for further reference.

This handbook is based on the series of articles that I wrote from '72 to '76 in the MATHEMATICS TEACHER (India), the Official Journal of the ASSOCIATION OF THE MATHEMATICS TEACHERS OF INDIA.

I would consider my efforts rewarded if teachers of mathematics find this handbook helpful in providing their pupils with club atmosphere for enjoyment of mathematics and enrichment of their mathematical background.

Teacher training institutions have all over the world started including maths club and its importance in their

syllabus and training programmes.

Suggestions for improvement and innovation are welcome and would be acknowledged in future editions.

—AUTHOR

"A LAMP can never light another lamp unless it continues to burn its own flame. The teacher who has come to the end of his subject, who has no living traffic with his knowledge but merely repeats his lessons to his students, can only load their minds; he cannot quicken them. Truth not only must inform but inspire. If the inspiration dies out and the information only accumulates then truth loses its infinity. The greater part of our learning in the schools has been wasted because, for most of our teachers, their subjects are like dead specimens of once living things, with which they have learned acquaintance, but no communication of life and love."

—RABINDRANATH TAGORE.

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1. WHY MATHS CLUB

EVERY teacher of mathematics knows that the class room situation is not conducive to catering to the needs of the gifted. The prescribed curriculum is such a poor fare that a lot of interesting and fascinating experiences that mathematics provides is bypassed on the plea that they are not relevant to the examination needs. By giving the talented pupils high marks for their performance in a test paper devised to make the average and the below average pupils trained on set questions and models to get through, teachers commit the crime of giving them a false picture about mathematics, if not their ability. Not only that; we do not throw open to them opportunities through challenging involvements to ensure their growth commensurate with their inherent potential. This leaves a country impoverished through non-exploitation of mathematical resources in pupils.

So professional integrity demands that mathematics teachers should join hands to set right this pathetic wastage of mathematical talent by providing a climate for fostering it in the school, which is possible only through a well-organised and well-run mathematics club. If the math club is to offer a varied and rich fare to members and should not at the same time tax the teachers, the best way to share the club activities is by taking charge of one or more of them according to exigencies of circumstances. The teaching time need not be affected, by holding club activities once a week in after school hours.

Mathematics belongs to the realm of fine arts like music, dance and painting. But this is yet to be conceded by every educated person. So the mathematics teacher can give opportunities to pupils through club programmes to get to know how mathematics can be enjoyed as a fine art. One could even boldly make the assertion that no other subject in the curriculum is so satisfying and self-fulfilling as mathematics with its pattern for discovery, its puzzles and fallacies, its exquisite language and logic, and its recreational and problem-solving aspects. Mathe-

mathematics admirably lends itself to cultivation of the habit of genuine research at junior level. The game spirit in mathematics provides real fun. Who can afford to forget that 'Mathematics is the King of all arts and the Queen of all sciences', a fact which is getting more and more appreciated in this computer age? Who will not admit that it is not just the age of Science and Technology but the age of Math, Science and Technology?

There is another important reason for running a mathematics club. The creative period in a man starts from his 14th year and generally lasts till his 25th year, as could be seen by the lives of most of the mathematicians and scientists. Unless pupils are exposed to the widening frontiers of mathematical knowledge and its applications while they are in their teens, they would not be favourably disposed to mathematics when they grow up. It would be worthwhile to quote Dr. Dubley of Woolwich Polytechnic in this connection. He writes in his book 'Development of Modern Mathematics' first published in 1970 that "more mathematical research has been done in the last twenty years than in the whole of the previous history of mathematics." We are living in the golden age of mathematics when axiomatisation has been resorted to in all its splendour in every branch of mathematics, algebra and geometry having become algebras and geometries and almost every branch of knowledge being mathematised.

One factor that might militate against the running of math club is the feeling on the part of some teachers, why, when they have become teachers in mathematics without having had anything to do with math club during their school days and even their college days, they should subject themselves to unnecessary burden through organising and running math club. One could only say that such a feeling displays the ignorance of explosion of mathematical knowledge that is characteristic of the post second world war era and indifference to the future of their wards. That feeling would be overcome when it is realised that more mathematics has come to mean more science which in turn means rise in standards of living. Teachers of mathematics cannot excuse themselves on any

plea from their responsibility of enabling children to face the present day fast changing world. They would have to organise study circles and devote their leisure time to study history of mathematics, modern topics in mathematics and mathematics for recreation and fun, besides acquainting themselves with better methods of teaching. They would not fail to see that the whole thing is highly rewarding and the pupils, the community and the State would look up to the teacher with regard and respect. The heads of schools should also allot a portion of library funds for buying mathematics books and journals of interest to teachers and pupils. Math club provides a teacher, the rare opportunity of enriching not only himself but also the youngsters through joyful exercise of the mathematical faculties with which nature has endowed them. The Government could help by giving special grants exclusively for mathematics equipment and having a place for math club activities in the annual inspection report.

Books for further Guidance:

1. Andrey Todd, *The Maths Club*, Hamish Hamilton, London.
2. Walter H. Carnahan (NCTM), *Mathematics clubs in High Schools*.
3. NCTM, *Handbook for Organisation of Mathematics Teachers*.

(NCTM: National Council of Teachers of Mathematics, 1906 Association Drive, Reston, Virginia 22091).

2. A BRIEF SURVEY OF MATH CLUB ACTIVITIES AT VARIOUS LEVELS

5

AN outline of the activities and programmes which could be had for profit and pleasure in school math clubs will speak for itself. Of course, the club programmes should reflect the findings of Piaget, Bruner and others and suit the levels of maturity attained by children.

Taking the *primary level*, the club programmes will naturally be activity oriented involving handling of real objects. Setting up patterns and discovering the patterns is most popular with children. For example, we can take cubes, cuboids, cylinders and even unfamiliar solids not included in the syllabus and ask the children to set up a pattern like cube, cube, cylinder, cuboid, cube, cube, cylinder, cuboid, etc. The scope is vast with centre of interest shifted to colour, shape, size and their combinations and this can well become a delightful item in every club meet. The objects may be replaced by triangles, rectangles, squares, circles, etc., and then by natural numbers. Children could be seen reaching heights not expected by teachers. Another area suited at this age is combinatorics. And we can ask children to count the number of squares of all sizes in a square grid, considering in succession 1×1 , 2×2 , 3×3 , etc., square grids. No formulas need be expected at this stage. Number of ways of exchanging a coin in terms of specified denominations with order and without order is another source of interest to children. This prepares them for the fascinating world of number partitions and Fibonacci numbers. We can also expose children to the elements of number theory and encourage them to look for discoveries by their own efforts like evenness of natural numbers, classifications of natural numbers into primes and composites, looking up for subsets of the set of natural numbers having interesting properties like those of the products of two consecutive natural numbers, the sums of two consecutive natural numbers, the product of the end numbers and its relation to the square of the middle number in every set of three

consecutive natural numbers, and so on.

Informal geometry gives plenty of scope to children to continue their flair for space exploration without geometrical instruments and spell out their discoveries. Discovery of incidence properties of lines (by using straight sticks), of planes (by using flat cardboard pieces), planes and lines, etc., is enjoyed by children. Topological properties of space such as open and closed curves interior and exterior of closed curves, Euler's topological properties and so on of solids and networks, central and axial symmetries of figures, and tessellations appeal to Primary School Children.

At the middle school level, children can be exposed to much richer fare. Patterns centring round reasoning in a rudimentary way can be attempted with finite sets and preparing them to see the need for proofs while handling infinite sets will be the natural follow up. Making statements regarding number properties and discovering them when they are false through counter-examples naturally go a long way in improving at this stage the level of mathematical maturity of children. Numeration systems to different bases, semi-axiomatic approach to number systems, modular arithmetic, properties of figurate numbers, divisibility patterns, properties in transformation geometry and properties of collinearity and concurrency, logarithms, etc., give much scope for children to do a lot of creative work on their own.

At the high school level, children could be helped to develop themselves to a high degree of mathematical maturity. The gifted can be allowed to finish their prescribed portions earlier and join advanced placement programme for study of special topics like statistics, probability, matrices, theory of games, linear programming, calculus, non-Euclidean geometry, Set theory, Boolean algebra, etc. They can be helped to have the satisfaction of reaching a certain stage in the study by sitting for an open book test intended to assess more of understanding and application than of memory. Children can be given junior research programmes; they can involve themselves in such activities on week ends and during vacations. Some research projects can be related

to traversability of curves, maxima and minima problems like ensuring the box to have maximum capacity when made with a cardboard with congruent squares removed at the four corners, patterns in the coordinates of the corners of different kinds of quadrilaterals drawn on a graph-sheet, drawing and studying advanced mathematical curves (algebraic and trigonometric), functions and compositions of functions, nomograms, etc. The mathematics club can also hold special courses on computers and on topics in modern algebra like groups, rings, fields vector spaces, and Boolean algebra, theory of equations, etc. Exhourses, meaning exhibition courses, in modern topics could be held and the brighter children be invited to study the charts put up in a sequence to reflect the growth of ideas on situation-abstraction-situation basis and take a test. This will go a long way in bringing to the notice of the inquisitive children, the thrill of introducing themselves to newer thinking in mathematics.

At the higher secondary level, i, complex numbers, vectors, tensors, analytical number theory, special functions and their uses, statistical project, mathematical logic, geometry in various transformations, flow chart and programming would constitute enough fare for study. presentation and discussion.

Some appropriate and interesting programmes for math clubs in *teacher training institutions* would be (1) to take up a topic put down for higher standard and devise a suitable method to teach it to the pupils of lower standard and (2) to invent instant improvisation techniques which involve use of waste stationery and materials available in the environment

The annual exhibitions and expositions form the most popular item in the programme of activities of a mathematics club. In exhibitions, exhibits are important and, instead of exhibitions, we could hold, for better impact on visitors, expositions where children vary situations and present concepts through dialogues and discussions. While holding expositions at the primary level, four batches of pupils should be trained to participate in the same item of the expositions, whereas at the middle school level in

exposition can be managed with only two batches. The high school exposition can, of course, be managed with just one batch. While preparing children for the exposition or exhibition, care should be taken to see that they are not trained through the method of telling and tutoring but through the method of 'enquire and find' or guided discovery method which alone could ensure a good grasp of the topic, enabling them to develop confidence to do 'on the spot thinking' and face the strangers and their questions successfully.

THE nature of organisation of Math Club depends on the strength of the school. If a school has more than five hundred pupils, it would be advantageous and congenial to have a Senior Wing and Junior Wing of the club respectively for classes IX onwards and classes VI onwards, as it would facilitate conducting of programmes suited to the levels of attainment of pupils. Opportunities should however, be there for members of one wing to participate if they so desire, in the programmes of the other. There is no need for any formulation of formal rules for the constitution of the club. In order to popularise the names of great creative mathematicians, the club and its wings may be named after them. Every pupil before leaving school should not fail to have heard at least the names of eminent mathematicians like Euclid, Fibonacci, Euler, Gauss, Leibnitz, Newton, Abel, Galois and Cantor. As there is a widely prevalent notion that mathematics is outside the reach of girls, clubs in girls' schools can be named after the five great women mathematicians celebrated in history, viz., Hypatia, Agnesi, Sophie Germain, Kowlevsky and Emmy Noether.

Membership:

If Mathematics were to involve paper and pencil or board and chalk work, running a math club is the least expensive. There is no need, therefore, for any subscription. If, however, it is desired to run it on the lines of a literary association with gala inaugural functions, special meetings and celebrations, valedictory functions and dramas when distinguished men are invited, payment of a nominal subscription per term or per annum may be made the minimum obligation for primary membership with slight variations to distinguish different cadres.

To inject a sense of belongingness, I have found that it is desirable to declare that not only is every pupil of the school eligible for membership in the club but also every pupil, by virtue of his being on the rolls of the school, is

an ordinary member of the club. Those who have constantly secured good marks in mathematics would be considered the primary members of the club, with a special pin or badge if it could be provided. That boosts their morale and secures their ready and willing participation. Once a member is enrolled in the primary ranks, it should be a point of honour with him or her to attend without fail, if not participate in the programmes arranged periodically by the club. So a club will have ordinary and primary members to make it open to all and give recognition to those with special aptitude. A sense of exclusiveness should at all costs be avoided to prevent friction and development of unhealthy spirit. As mathematics is an all embracing branch of knowledge, the club should also preserve that character of embracing all those interested in its fold. It is useful in more than one way to have old boys who have left the school within five or ten years as honorary or donor members. It is also essential to canvas and maintain a cadre of resource persons mostly in the neighbourhood of the school to give opportunity for members of the club through access to experts for preparing their programmes and doing small scale research. They may also act as examiners or judges in the competitions that may be held annually. Well-to-do persons or public spirited firms can be approached to serve as patrons for bearing the costs of subscription for journals of mathematical interest, the folders for which may bear their names as donors.

Office-Bearers:

Besides the Head serving as ex-officio president and a mathematics teacher to serve as Teacher Secretary or Guide, capable and trustworthy pupil members should be given ample opportunities to take real responsibility in running the affairs of the club in various capacities as student president, student secretary, student treasurer, member of the working committee, etc., and in maintaining regularly records of the activities and transactions of the club. This would evoke leadership qualities developed through exercise and discharge of duties to a com-

munity. The Teacher Secretary should find time and energy to ascertain the needs of members of the club if they are articulate enough or to help them in discovering their needs and to guide them with genuine enthusiasm and abiding interest. It should not be the case of doing something merely to please or placate the inspectorate...or the management, whenever the pressure is felt. There could be joint secretaries, one from top most standard and the other from the next lower standard in each level, as it would give an opportunity for the latter to continue as one of them the next year. This has been found to provide continuity and stability to the club. Even a top most standard boy who has just left the school can be asked to continue as honorary secretary representing old boys at least for a year. His presence at important functions gives him and others a great satisfaction and he can be seen to provide guidance to his juniors.

The Secretaries in particular should be chosen from among those who have not only aptitude in mathematics but show flair in moving with others with courtesy and interest. He or she should see to it that all talented members get some opportunity or other every year to exhibit and develop their talents. He should evince great interest in running a bulletin board and in giving announcements through the P.A. system in school to see that every member is reached regarding club activities and programmes.

The Student President should be given adequate opportunity to preside over the club meetings and conduct the proceedings. It is essential that staff members of the school should opt to be in the background and encourage the pupil office-bearers in all healthy ways to put up their own show in their programmes. It is assumed that pupils will not be simply tutored to serve as mere shadows of teachers or parents. To develop pupils is of course time consuming but rewarding as they need revisions and rehearsals for at least a week prior to the holding of a club programme.

Books for further Guidance:

1. Samuel Jones, *Mathematical Clubs and Recrea-*

- tions'*, *'Mathematical Wrinkles'*, *'Mathematical Nuts'*. S.I. Jones Co., U.S.A.
 2. Andrey Todd *The Maths Club*, Hamish Hamilton, London.
 3. Walter H. Carnahan (NCTM), *Mathematics Clubs in High Schools*.
 4. *Handbook for Organisation of Mathematics Teachers*, (NCTM).
- NCTM: (National Council of Teachers of Mathematics, 1906, Association Drive, Reston, Virginia 22091).

4. ACTIVITIES-MEETINGS

(A) Meetings:

NO club can be considered to function without its periodical meetings. Unless members are highly motivated, it will be difficult to hold weekly or fortnightly meetings. So monthly meetings will be found convenient. Without adequate preparation and one or two rehearsals, the participants do not give a good account of themselves. The programme of meetings usually comprises of the inaugural function, ordinary meetings every month, one or two special meetings and the valedictory function. Eminent professors of mathematics from colleges can be invited to speak at the inaugural and valedictory functions. The entire function should be gone through by the school pupils themselves, except for the presidential address from the visiting professor, the Head, or any interested member of the Managing Board or Parent Teacher Association. As inaugural and valedictory functions hold the excitement of speaking in the presence of outsiders, bright boys and girls should be encouraged to take up important themes and give talks on these at depth suited to their levels of maturity. Some of the themes which have been successfully presented as viewed from my experience over the years are the following:

1. Algebra — as X-ray arithmetic.
2. Development of Number Systems — Semi-axiomatic approach.
3. Figurate Numbers and their properties.
4. The language of line.
5. The revolution in geometric thought.
6. The Set Language.
7. Mathematical Systems — group, ring, field, vector space, Boolean algebra, lattice, etc.
8. Modular Arithmetic and its structural properties.
9. Rationale of short cut methods.
10. Pythagorean triplets.
11. Prime Numbers as building blocks of arithmetic.

12. Magic in magic squares.

13. Fibonacci Sequence.

Black board work needs to be done by a supporting pupil member or members with good handwriting, while the pupil speaker gives his expository talk.

Monthly Meetings:

Monthly meetings can be presided over by mathematics teachers, non-mathematics teachers handling particularly science, engineering and craft subjects or the pupil chairman himself. To provide items of interest to different levels of ability, the programme for a meeting may consist of (1) a talk about the life and work of a mathematician, (2) number games for participation by the members of the audience, (3) demonstration talk on a theme from geometry, mensuration or trigonometry, (4) some application of mathematics in industry, sports, etc. (5) fallacies and paradoxes and (6) distribution of mathematical paper objects to rouse and sustain the interest of the members in the club meetings. Flannel board is an excellent visual aid with which transformations of various kinds in geometry, algorithms, etc., can be taken up in a club meeting for treatment to a greater depth.

For example, pupils learn $\sum_{i=1}^n n = \frac{n(n+1)}{2}$ in core mathematics. This can be used in finding $\sum T_n$ where $T_n = a + (n-1)d$. Use of identities in finding $\sum n^2$ and $\sum n^3$, number properties through identities, different proofs of the Pythagoras theorem; volume of a bucket (frustum of a cone) and of tennikoit ring (solid ring of circular cross section), properties of Pythagorean triplets; different kinds of averages, number of diagonals of a polygon, extension to synthetic division by a trinomial, binomial expansion for the general case with positive, negative and fractional indices, continued fractions, solving riders by vector methods, and using $\left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 = ab$ in establishing $\left(\frac{a+b}{2}\right)^2 + \left(\frac{c-d}{2}\right)^2 = \left(\frac{a-b}{2}\right)^2 + \left(\frac{c+d}{2}\right)^2$ when $ab=cd$ and getting the underlying number property etc., are some of the items

that have been taken up at the meetings organised under my guidance. Junior level research findings can also be included in the programme. They usually involve problems centring round combinatorics, maxima and minima problems in mensuration, solving of equations and generalisation of problems. The number on a railway ticket, a recreational problem appearing in a newspaper or magazine, pattern of numbers in a score board or a marksheet, etc., can serve as starting points for junior level research.

Special Meetings:

When opportunities are favourable, persons conversant with interesting applications of mathematics, say in music, dance, fine arts, designs, sports, architecture, prosody, survey, etc., can be invited to speak to the club members with realistic illustrations and demonstration to enrich the background of the members and to provide an insight into the versatility of mathematics as a tool. Vacating games can be played on the playground so as to bring out the underlying formulae like $S = 2^n - 1$ or $3^n - 1$. Three places are marked in circles on the ground. To start with, two or three players of varying heights, (no two of them being of equal height) may be asked to occupy together one place and then vacate it and go to any of the other two places by following the rules: (1) only one should move from one place to another at a time, (2) the taller one should not go into a place occupied by a shorter one (but not the other way), (3) the vacating should be done in the least number of steps which will be discovered to be given by the formula, $S = 2^n - 1$; if the order of movement is stressed in vacating, the least number of steps will turn out to be $3^n - 1$, n denoting the number of players in each case. Pupils can be asked to assemble in ten (equally spaced), rows of ten (equally spaced) persons each. Each of them can be given a mathematical name according to his or her seat number and row number. In naming games pupils satisfying equations such as $S = R$, $S + R = 5$, $S + R = 6$, $S - R = 1$, $S - R = 2$, etc., can be asked to stand up. The set of pupils satisfying both the equations $S + R = 5$ and

$S - R = 1$ is seen as a singleton (single element set). Inequations such as $S > R$, $S + R > 5$, $S + R < 10$, $S - R > 2$ etc., can also be taken up for commands. By making use of 'and' or 'or' combinations, intersection and union of sets and the meaning of simultaneous open sentences can be effectively brought home.

Flag drill to bring out number properties such as $1 + 3 = 2 \times 2$, $1 + 3 + 5 = 3 \times 3$, $1 + 3 + 5 + 7 = 4 \times 4$, $1 + 8 = (1 + 2)^2$, $1 + 8 + 27 = (1 + 2 + 3)^2$, $1 + 2 + 1 = 2 \times 2$, $1 + 2 + 3 + 2 + 1 = 3 \times 3$, $1 + 2 + 3 + 4 + 3 + 2 + 1 = 4 \times 4$, etc., can form an attractive programme.

Birthdays of mathematicians can be celebrated. A few scenes from their lives can be enacted to make the spectators relive the wonder and excitement that characterised the creative contributions of the mathematicians as revealed by their encounters. For example, Ramanujan's encounter with his II Form mathematics teacher, Gauss' encounter with his III class mathematics teacher and Euler's solution of Koenigsberg's bridge problem, Euclid's proof of the existence of five and only five platonic solids, etc., have a dramatic appeal. If enactment is found inconvenient, boys with simple make-up or mask to resemble mathematicians as seen in their pictures can give in first person a narration of their lives and achievements. Imaginary interviews can also be presented.

In big schools the need for junior wing programmes will be felt and pupils of the senior wing can be given opportunities to speak in the junior wing meetings. Junior wing programmes are found to be popular if fun with figures, pattern discoveries, story-problem building, anecdotes from the history of mathematics, mathematics in games, etc., are made the major items.

Mathematical duets and duels can be arranged once a while to inject a spirit of competition between two top fellows in different classes or to present the mathematical fallacies and paradoxes in a telling fashion.

Day long mathematics seminars and symposia interspersed with mathematical doodling could be arranged once a quarter, on a Saturday, if there is enough enthusiasm and involvement in the subject matter on the part of mem-

bers. Parents and others including pupils and teachers of other schools could be invited to be observers. Participation should be expository or research-oriented, with a lot of demonstration material. The object is to provide an atmosphere for the members to get exposed to the spirit of mathematics and its unique features as differentiated from the other pursuits of knowledge, especially science.

Books for further Guidance:

1. E.T. Bell, '*Men of Mathematics*' Vols. I and II (Pelican) Penguin.
2. Northrop, '*Riddles in Mathematics*' (Pelican) Penguin.
3. W.W. Sawyer, '*Mathematician's Delight*', '*Prelude to Mathematics*', '*Vision in Elementary Mathematics*', '*Patterns in Mathematics*' (Pelicans) Penguin.
4. Irwing Adler, '*New Mathematics*', '*A New look at Arithmetic*', and '*A New Look at Geometry*' —Mentor.
5. W.W. Sawyer, '*Concrete Approach to Abstract Algebra*', (Freeman).
6. E.A. Maxwell, '*Gateway to Abstract Algebra*' —CUP.
7. P.K. Srinivasan and M.C. Gupta, '*Discover Patterns and Enjoy Mathematics*', Frank Bros, Co. New Delhi.
8. Z.P. Dienes, '*Mathematics through the senses, games, dance and art.*' (NFER—UK).
9. Gardner: '*Mathematical Puzzles and Diversions*', '*More Mathematical Puzzles and Diversions*', '*Many more Mathematical Puzzles and Diversions*', '*Mathematical Carnival*'.—(Pelicans) Penguin.

5. ACTIVITIES-QUIZ PROGRAMME

WEEKLY or fortnightly quiz programmes are popular with bright students. Quiz programmes do not need any preparation except for the resourcefulness of the quiz master. In a big school, the programmes could be held at two levels, junior and senior.

The results of each quiz programme should not only be announced on the notice board but also be maintained in a record book for final declaration of the year's quiz champions and runners and award of certificates or prizes.

The quiz programme could be conducted either by a written paper or by oral questions, depending upon the atmosphere of the school. While conducting a quiz programme, a non-participating member of the club may serve as the recorder.

If a question is answered fully, the first person or the first set of persons who answers it gets full points, the points being divided equally among the persons of the set. If it is answered partially, the partially correct answers will be given equal fractional credits. If a question is not answered, the quiz-master, who has to give the answer will be considered to have scored the point for the question. To warm up, certain questions may be given without scoring of points. That would help the quiz master to assess the level of knowledge of the participants. It would help him to spot out the most well-informed participants.

The object of the quiz programme is to encourage acquisition and command of mathematical facts, rather than skills. A quiz programme should not be confused with mental arithmetic. A few questions on pattern discovery may also find a place in the programme.

It is a common sight to see more spectator members than participant members gathered for the programme and all of them should be given credit of attendance through collection of membership cards or entry slips. If a school has a tape recorder some of the programmes can be recorded and broadcast for the benefit of the school community once a quarter, to create interest in and appre-

ciation for mathematics and to make the programmes popular.

A quiz programme should be brief, lasting not more than forty five minutes and about 10 to 12 questions should be put. This would avoid strain on the part of the master and participants.

Two typical programmes, one at Junior level and the other at Senior level are given below with answers for guidance.

Junior Level Quiz Programme

1. Which number is divisible by all numbers? (0)
2. Which number divides all numbers? (1)
3. Which is the first prime number? (2)
4. Which is the first composite number? (4)
5. What is 20 per cent of 20 cent? (4 per cent)
6. Give $1/2$ per cent as a fraction? ($1/200$)
7. Express 0 as a fractional number (0/1, 0/2, etc.)
8. Express 1 as a fractional number. ($1/1$, $2/2$, etc.)
9. Give the first whole number which is equal to the sum of its proper factors. ($6 = 1 + 2 + 3$)
10. Express 2 as percentage. (200 percent)
11. Name the triangle with one angle a right angle and two sides equal. (isosceles right-angled triangle)
12. By what number should you multiply 8 to get 5? ($5/8$)
13. What number when added to a second number leaves the second number un-

- changed in the sum? (0)
14. Is the sum of two odd numbers odd or even? (even)
15. If A gets $1/3$ more than what B gets, by how much does B get less than A? ($1/4$)
16. Give a fractional number between $1/4$ and $1/2$. ($1/3$, $3/8$, etc.)
17. If at intervals of 10m posts are erected, how many posts will be erected along a route 1 kilometre long? (101)
18. If a clock takes 22 seconds to strike 12, how long will it take to strike 6? (10)
19. A circle is inscribed in a square of side 10 cm. What is the radius of the circle? (5)
20. What is the minimum number of lines needed to form a closed figure? (3)
21. How many straight lines pass through two points? (1)
22. If two complementary angles are equal, what is the magnitude of each? (45°)
23. Are two equal sets equivalent? (Yes)
24. How many subsets does a set of 3 elements have? ($2 \times 2 \times 2$)
25. Which set has only one subset? (the empty set)

Senior Level Quiz Programme

1. Give the common factor of two whole numbers relatively prime to each

- other. (1 only)
2. What is the L.C.M. of two numbers relatively prime? (their product)
3. The L.C.M. and the G.C.D. of two numbers are 24 and 4. If one of the numbers is 12, what is the other? (8)
4. Why is 1 neither prime nor composite? (a prime number has only two factors and a composite number has more than two factors. But 1 has only one factor)
5. Mention the names of quadrilaterals which are cyclic (square, rectangle, isosceles trapezium and quadrilateral with opposite angles supplementary)
6. Mention the names of quadrilaterals having their diagonals equal. (square, rectangle and isosceles trapezium)
7. Mention the names of quadrilaterals having their diagonals cutting at right angles. (square, rhombus, kite, quadrilateral with diagonals cutting at right angles)
8. If the numbers giving the surface area and the volume of a cube are equal, what is the edge of the cube? (6)
9. What is a perfect number? (a number equal to the sum of its proper factors)
10. What is the second perfect number? ($28 = 1 + 2 + 4 + 7 + 14$)
11. What is the tenth prime number? (29)
12. If the gain is 20 per cent of S.P., what is the true percentage of gain? (25 per cent)
13. What is the first square

- odd number? (1)
14. What is the first even whole number? (0)
15. Give the first number which can be expressed as a sum of two cubes in two different ways. ($1729 = 10^3 + 9^3$ or $12^3 + 1^3$)
16. Express 17 using four 4's. ($\frac{4}{4} + 4 \times 4$)
17. Is $-x$ positive or negative? (depends on the value of x)
18. Is $-x^2$ positive or negative? (always negative)
19. What is wrong in saying that $\sqrt{2} = 1.414$? (1.414 is only an approximate rational value for $\sqrt{2}$; $\sqrt{2}$ is an irrational)
20. What is wrong in saying $\pi = \frac{22}{7}$? ($\frac{22}{7}$ is only an approximate rational value of π ; π is an irrational number)
21. If a square is inscribed in a circle of diameter 10 cm, find the area of the square. (50 sq. cm)
22. How many lines of symmetry does a square have? (four)
23. If $x^2 = 4$, what is x ? (± 2)
24. In what domain can $a^2 + b^2$ be factorised? (in complex number domain: $(a + ib)(a - ib)$)
25. What is the solid formed when a right-angled triangle is rotated about its hypotenuse? (double cone)

Books for further Guidance:

1. Heafford and Babb — *A New Mathematics Fun* — A Quiz Book — Hutchinson of London.
2. P.K. Srinivasan and M.C. Gupta — *Discover Patterns and Enjoy Mathematics* — Frank Bros and Co., Chandni Chowk, New Delhi 110 006.

AMONG the activities of the mathematics club, running a bulletin board is of basic importance. It can even be said that it is the absolute minimum that should be done to keep the club alive throughout, every year. This programme does not involve much of organisational strain as is the case in arranging meetings, exhibitions, etc.

The birthdays of mathematicians can be celebrated at no cost but very effectively by displaying their pictures and putting up special write-ups on their lives and contributions.

Clippings of mathematical interest from magazines, Sunday supplements and newspapers can be put up periodically to direct the attention of children to items of mathematical interest appearing now and then in the dailies and other periodicals.

Alternative proofs for geometric theorems and riders found by the pupils can be given due publicity on the bulletin-board to give a sense of satisfaction and self-fulfilment to the discoverers and to induce other pupils to emulate and excel them if possible.

Curve stitching and geometric designs in colour can be displayed to attract attention and make mathematics appealing to the eye and the imagination.

Problems for solution can be proposed by gifted pupils or teachers periodically. Good solutions can be duly announced from time to time. Programmes for discovery of patterns in number theory, geometry and algebra can be published fortnightly once. Junior research problems or projects for vacations can be announced in advance and outcomes of the projects made known after the school reopens.

Elegant answers in the examinations and tests and class room work can be displayed from time to time to give recognition to pupils offering such answers and to make others develop a taste for appreciating such answers.

Under the caption 'Enjoy the Fun of Learning New Things,' interesting items from Mathematics Periodicals

or Enrichment Mathematics Books can be given space on the bulletin board. Particulars of useful mathematics books added to the library or available with the local book-sellers deserve to be given due publicity.

Notices about meetings, quiz programmes, competitions and exhibitions, etc., will of course find place on the bulletin board. Competition results and names of quiz champions for each year will also be announced on the board.

A thought for the week taken from the utterances of great mathematicians can be put up on the bulletin board.

A few samples of problem solving, discovery programmes and junior research projects are given below for guidance:

Problem solving: 1. From any point in an equilateral triangle, draw perpendiculars to the sides. How is the height of the equilateral triangle related to the three perpendiculars?

2. The number of units in the area of a sphere is equal to the number of units in its volume. What is the radius of the sphere?

3. Given three non-zero digits how many two digit numbers can be formed? How many three digit numbers can be formed?

Discovery Programme: 1. Take the square grids 1×1 , 2×2 , 3×3 , 4×4 , etc. Count all the squares, big and small in each, discover the pattern in the counts and give the total number of squares in a 10×10 square grid.

2. Take wooden cubes $1 \times 1 \times 1$, $2 \times 2 \times 2$, $3 \times 3 \times 3$, $4 \times 4 \times 4$, etc., painted on all sides. If they are cut into unit cube pieces, how many of them will have paint on (i) three sides, (ii) two sides, (iii) one side and (iv) no side, in each case?

3. Take any four whole numbers, say, a, b, c and d . Derive four other numbers by finding differences between a and b , b and c , c and d , d and a . Repeat this process and see what happens to the differences after a few stages.

Junior research projects: 1. Finding volumes and surface areas of solids of revolution got by rotating a right-angled triangle (i) about an edge and (ii) about the hypo-

tenuse.

2. Discovering three-dimensional analogues of plane geometry theorems.

3. Number of paths on a grid from one corner to another.

4. Modular arithmetic on a sheet calendar.

5. Numeration systems of base 2, 4, 8 and 16 and instant conversion from one base to another.

6. Proofs in number theory starting with proofs of: (i) 'Product of two consecutive whole numbers is even' and (ii) Product of three consecutive whole numbers is divisible by 3'.

7. Connection between the width of the pathway all round a rectangular strip of paper and the capacity of the box formed by removing the four square corners and raising the flaps to form a box.

A sample proof is given below to show what our pupils are capable of, if directed properly and how they can develop a taste for mathematical proofs and the role of counter-examples.

Statement: The product of any three consecutive whole numbers is divisible by 3.

Proof no. 1: Taking any three consecutive whole numbers, the first number can either be a multiple or a non-multiple of 3. If it is the former, then the statement is proved.

If the first number is not a multiple of 3, then it leaves either 1 or 2 as remainder when divided by 3.

If it leaves 1 as remainder, then it is of the form $3n + 1$. Then the two consecutive numbers are $3n + 2$ and $3n + 3$, $3n + 3$ is divisible by 3. That is to say, the statement is true.

If it leaves 2 as remainder, then it is of the form $3n + 2$. Now the next number becomes $3n + 3$ and it is divisible by 3.

Hence the product of 3 consecutive whole numbers is always divisible by 3.

Proof no. 2: Assume the statement is not true. Then the statement that the product of any three consecutive whole numbers is not divisible by 3 should be true. But

consider the counter-example. $4 \times 5 \times 6$ is divisible by 3. So our assumption is false. Hence the given statement is true.

Books for further Guidance:

1. *Net work - A Mathematics Series* — Hutchinson and Co. (Publishers) Ltd. — U.K.
2. *Mathematics Curriculum — A critical review* (8 books). Blackie and Son Ltd./School Council.
3. Teri Perl: *Math Equals - Biographies of Women Mathematicians and Related Activities* — Addison — Wesley (USA)
4. NCTM (USA) — *Historical topics for the Maths Classroom* — 35th year book.

ONE of the serious complaints levelled against the educational atmosphere maintained in schools is the absence of sufficient challenge in the fare provided for or in the work assigned to the gifted pupils and the consequent poor level of aspiration on their part. This situation needs to be remedied by giving them opportunities to develop their creative potential through engagement in small scale research programmes spread over two or three months including vacations. As most of the parents are marks conscious, efforts of an enthusiastic teacher in this direction may be misconstrued by the parents as 'waste of time'. In my experience I have found that one way of meeting this psychological handicap is to allow the gifted pupils to finish the prescribed portions earlier, particularly non-language subjects. Of course the Head should be told of this and his/her consent obtained to avoid difficulties that may be created by misunderstanding colleagues. Most of the gifted pupils do welcome this programme as it gives them a sense of importance and a satisfaction of accomplishment. The most successful participants in this programme feel it an honour to be allowed to present their research findings in a specially invited meeting of the teaching staff towards the end of the school year, when each participant is introduced by a member of the staff and receives acclamation for his performance and achievement, from the assembled teachers.

Without instituting such a programme, organisation of mathematics club, in my opinion is incomplete. To imagine that one would become research conscious only when one goes for a Ph.D. programme is a myth and it is exploded by the lives and achievements of mathematicians and scientists. The breaking of adolescence is the signal for quest for new directions and insights and for indulgence in experimentation out of sheer curiosity and sense of adventure. This ferment tapers off by about one's twenty-fifth year in most cases. Almost all the ingredients that go

into research output in one's life sprout out and become tangible during this period.

To prepare gifted children for the advent of this creative phase, they should be exposed to recognition and discovery of patterns, abstraction of a principle underlying or central to certain types of situations and building models to concretise principles right from the first day of their school.

So the small scale research programme will necessarily be planned and provided for at all levels sub-junior, junior, senior and super-senior. The programme for the lower level consisting of sub-juniors and juniors will consist mostly of pattern recognition and discovery, story building and model building and number compositions whereas the programme for the upper level consisting of seniors and super-seniors will embrace generalisation, extension, problem solving, number theory, investigation, explanation of mathematical principles involved etc. Upper level research participants should be encouraged to give proofs for their findings and to help them succeed, they should be made familiar with the well-ordering principle. The mathematical induction, the principle of infinite descent, the technique of deduction through truth tables, the method of exhaustion, the *reductio ad absurdum*, proof by the contrapositive etc. For students who are diffident but interested, their research may centre round applications of mathematics in industries or the explanations of the applications of mathematics in sciences, commerce and engineering.

I shall try to delineate some of the successful programmes launched under my guidance.

Lower Level: Pattern recognition starts with, say, finding more terms in given sequences. A few examples are given below:

1. 5, 7, 12, 19, ...
2. 1, 3, 4, 15, 64, ...
3. 1, 101, 1001, 10001, ...
4. 1, 2, 6, 24, ...

It may also be had in pairing game as, for example, finding the other elements in the incomplete pairs in the

sequences:

(1, 3), (2, 6), (4, 12), (6, -), (-, 24), (3, -)

(1, 4), (2, 9), (5, 36), (4, -), (-, 81), (7, -)

Children learn to do a lot of problems given in the text-book alone. If the trend is reversed and the children play building games, they can have lot of fun building story problems themselves, suited to the level of their attainment. Take for instance the equality $5 \times 8 - 3 \times 6 = 22$. The story problems can take various forms to concretise this abstract fact. A few are given below: There are 8 boxes of 5 red balls each and 6 boxes of 3 blue balls each. There are twenty-two more red balls than blue balls (or)

8 books cost on an average 5 units of money per book. I sell 3 of them at 6 units of money a book. To get back at least the amount of investment, for how much money should I sell the rest of the books? The sentence would then be given in the form $5 \times 8 - 3 \times 6$? A lot of ingenuity needs to be displayed in constructing story problems. Even adults have a tough time in problem-making.

Series of multiples of 2, 3, 4, etc., can be displayed in 10 x 10 square ruled sheets with the squares numbered from 1 to 100, by shading the squares corresponding to multiples of 2, 3, 4, etc. One gets beautiful designs this way.

Another interesting model is obtained by taking two square sheets 10 x 10 and erecting (i) on each square of one of these a rectangular block whose height represents the sum of the row number and the column number meeting at the square, and (ii) on each square of the other a rectangular block with height representing the product of the row number and the column number meeting at the square.

Children enjoy making their own designs from coloured paper bits of different but identical geometric shapes. This can be a free play leading to discoveries about tessellations of different geometric shapes.

Another interesting item is number composition, given a dot diagram. An example is given below:

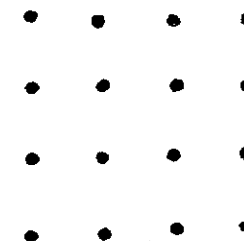
By partitioning the diagram in different ways one gets besides the ordinary ones like $4 \times 4 = 16$, $8 \times 2 = 16$, etc., the following number facts:

$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 4 \times 4$$

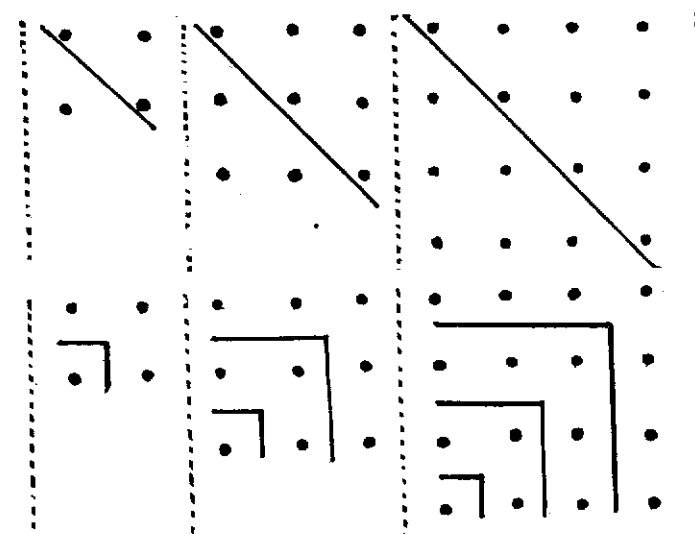
$$1 + 3 + 5 + 7 = 4 \times 4$$

$$6 + 10 = 4 \times 4$$

By giving a series of similar dot-diagrams, some general properties can be discovered. For example, consider the dot-diagrams:



We discover that



(1) the sum of two consecutive triangular numbers is a square number.

(2) the sum of consecutive odd numbers, starting with 1 is a square number.

Plotting games develop a good insight if the graph sheets are ruled in different ways, square-ruled, parallel-ruled, ruled at different angles, triangle-ruled, etc.

Paper dissection to change shape without change in area is not only a good pastime but also an exercise in developing spatial intuition.

Children find discoveries in combinatorics most exciting. For example, the following are good items: the number of beads required to represent two-digit numbers, three-digit numbers, etc., on spike abacus: the number matching figures that can be drawn in a vean diagram re

presenting two sets, of m elements and n elements; and the number of ways in which three or four circles can be drawn so as to be related to each other in different ways.

Upper Level: Some of the interesting investigations suited to this level are: (1) explaining short cuts in arithmetic, through algebra; (2) guessing properties of two consecutive numbers, three consecutive numbers, square numbers, prime numbers, cubes, partitioning, factorisation, etc., and proving them; (3) maxima and minima problems such as (a) the length of edge of congruent corner-squares to be removed from a rectangular sheet so as to make the remaining portion folded into a rectangular box of maximum capacity; (b) finding the shape of the plane figure having the maximum perimeter for the given area; (4) counting the number of triangles, squares, etc., that can be drawn in a dot diagram with a specified number of dots; (5) the correspondence between the amount of rotation from positive direction of the X-axis and the vectors obtained by changing the order and signs of the co-ordinates of the terminal point (a, b) of the starting vector making a given angle with the positive direction of the X-axis; (6) traversability of networks in the Euler and Hamiltonian senses; (7) finding solid geometry analogues to theorems in plane geometry.

Present day Mathematics Programmes carry quite a lot of interesting items for small scale research. Some of them are: (1) modular arithmetic and proving divisibility rules, (2) structural properties of modern arithmetic for prime number and composite number modulus, (3) numeration to different base systems, (4) discovery of logarithms of prime numbers by crude approximation method, (5) negative indices in binomial expansion through division approach, (6) finding formulas for the sum of divisors of a given number, (7) continued fractions, (8) group structures underlying transformation and permutation groups, (9) collecting recreational problems and proving them, (10) study of theorems in geometry in different sequences, (11) construction of riders, (12) examining the truth of statements and of their converses, inverses and contrapositives, (13) probability games and

graphs, (14) proofs of geometrical theorems and riders through vector methods, and so on.

For students, who have an interest in knowing the applications of mathematics, the research programme can centre round finding out the applications of the principles of mathematics in trades like carpentry, goldsmithy, printing, tailoring, spectacles-making and fitting, plumbing, in the making of pulleys, gears, watches, transistors, telephones, etc., in foreign exchange and in fine arts like music, dance, sculpture, painting, etc. They may be asked to build up tables giving the amount after n years at compound interest, by using the formula $A = P(1 + i)^n$ and to interpret algebraic relations. For example, take Pni ; it can be viewed as $(Pn)i$ or $P(ni)$. This equality interpreted gives that the simple interest on P units of money for n years at 1 per unit of money per annum is the same as the interest for one year on n times the principal at one unit of money per annum or the interest for one year at the rate of ni per unit of money per annum on the principal of P units of money.

Problem cards can be distributed on a week end day every quarter to help the pupils engage themselves in small scale research through group discussion and encounter. Red, green and pink problem cards of the Nuffield can be taken for the model.

Annual manuscript number containing research findings by students can be brought out and released at a public function. The number can afterwards be sent to the school library.

Books for further Guidance:

1. *Hungarian Problem Book I, II*, Singer, New York, 1963.
2. D.C. Shklarsky et al *The USSR Olympiad Problem Book*, Freeman, San Francisco, 1962.
3. S. Straszewicz, *Mathematical Problems and Puzzles from the Polish Mathematical Olympiad*, Pergamon, 1965.

4. J.N. Kapur — *Suggested Experiments in School Mathematics Vol. 1, II and III*, Arya Book Depot, New Delhi.

Periodicals:

1. Mathematical Gazette, UK.
2. Mathematical Magazine, USA.
3. Mathematics Teacher, INDIA.
4. Mathematics Teacher, USA.
5. Student Mathematics, CANADA.
6. Mathematics in School, UK.
7. Eureka, CANADA.

THE annual mathematics contests to spot out, recognise and foster mathematical talent should become a normal feature of schooling like terminal examinations. Contests can be held at primary, middle, high and higher secondary school levels in all schools. At the primary level, questions would centre round listing possibilities and combinations, building story problems around given mathematical sentences, closed and open, giving number compositions from dot-diagrams, paper folding and paper dissection. At the middle level, non-routine problems arising out of mathematically significant situations, pattern discoveries, etc., may be asked. At the high school level, the questions should not be based on worked models and other exercises already seen in the prescribed books or worked out in class. Questions to evoke creativity are appropriate at this level. Grasping a new idea through new symbolism and applying it to new situations should be given an important place in the contest paper at this level. Detecting fallacies in arguments, disproving statements, logical problems and problems of elementary number theory have their own place in the high school contest.

As mathematics contests are not yet common in schools of developing countries, some sample non-routine questions with not much complexity are given below for guidance to teachers who want to conduct contest for the first time. Once the participants develop confidence in facing questions such as these, the standard of the contest can be raised to any degree that is desired.

SUB-JUNIOR

1. Read as many number facts as you can from the dot-diagram given below:

.	.	.	.	.
.	.	.	.	.
2. Construct two stories for the following number facts:

.	.	.	.	.
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 - i) $8 \times 5 - 6 \times 5 = 2 \times 5$
 - ii) $\frac{6 \times 8}{2} = 24$

.	.	.	.	.
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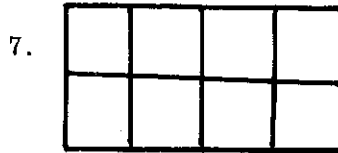
3. Using the three digits 3, 6, 0, list all the two digit and three digit numbers in ascending order.

4. The diagram given below is $\frac{4}{3}$ of a whole. Draw the whole.



5. I am 5 years junior to my elder brother who was born in 1950. I am senior to my younger brother by 2 years. When was my younger brother born?

6. If a three digit number is divisible by 11, find the relation among the digits.



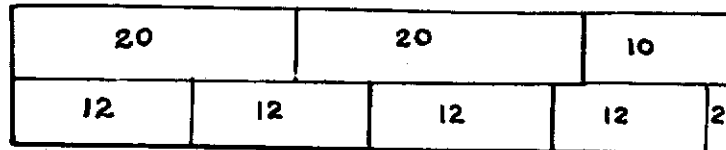
How many 1 cm sticks are required to build this net work?

8. 2 pencils and 4 pens together cost 20 units of money. What is the cost of 3 such pencils and 6 such pens?

9. How many times do we use 8 while writing numerals from 1 to 100?

JUNIOR

1. Read as many number facts as you can from the diagram given below:



2. Construct two story problems for the following sentences:

(i) $8x + 2 = 26$

(ii) $x \div 3 = 9$

3. Find the minimum number of rectangles each measuring 8 units by 6 units you will need to build a square with them.

4.



- If AB is 2 units, find AD.
- If AD is 1 unit, find AB.

5. Draw the various ways in which three circles can lie with respect to each other.

6. Register numbers of pupils going for a public examination in a school were given from 400 to 748 both inclusive. While examining the numbers it was found that 517 was given twice, and 665 was written next to 644. If the errors had not been made, what should have been the register number of the last boy of the school?

7. Write 1 as a sum of three unequal fractional numbers.

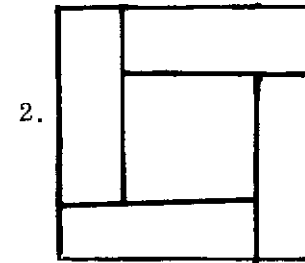
8. There are four planks in a wooden shelf. The second plank has 5 books less than the first shelf. The third plank has 5 books more than the second shelf. The fourth plank has 5 books more than the third. Which planks have equal number of books? How many times the number of books in the first plank is the total number of books in the shelf.

SENIOR

1. Construct two stories for the following number facts:

(i) $(-8) - (-11) = +3$

(ii) $(-5) \times (-2) = +10$



How will you use this diagram to prove Pythagorean property of a right-angled triangle?

3. Point out the lack of precision in the language used in the following questions:

(i) What should be added to $a^2 + 6a$ to make it a perfect square?

(ii) What is the smallest number by which 252 should be multiplied to get a perfect square?

4. Factorise: (i) $(x-2)(2x+3) = (x-2)(x-1)$; $x \in \mathbb{Z}$

(ii) $(3x-5)(4x+5) = (3x-5)(4x+3)$; $x \in \mathbb{Q}$

5. Interpret geometrically the following:

(i) $a^2 = b^2 + c^2$

$$(ii) \frac{\pi}{4} a^2 = \frac{\pi}{4} b^2 + \frac{\pi}{4} c^2$$

$$(iii) \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} b^2 + \frac{\sqrt{3}}{4} c^2$$

6. Show that in a cone the base area can never be equal to its curved surface area. Examine whether the base area will be less or greater than the curved surface area.

7. If $ps - qr$ can be represented by $\begin{vmatrix} pq \\ rs \end{vmatrix}$

$$\text{show that (i) } \begin{vmatrix} rs \\ pq \end{vmatrix} = - \begin{vmatrix} pq \\ rs \end{vmatrix}$$

$$\text{and (ii) } \begin{vmatrix} pr \\ qs \end{vmatrix} = \begin{vmatrix} pq \\ rs \end{vmatrix}$$

8. What are the various ways of arranging 6 rectangular boxes each measuring 5 units by 3 units by 2 units. Assume that each arrangement is made in such a way that identical faces of any two of them overlap each other perfectly. Find the total surface area of the exposed faces in each arrangement.

SUPER SENIOR

1. Define the distance between two points (x_1, y_1) and (x_2, y_2) in a co-ordinate plane as $|x_1 - x_2| + |y_1 - y_2|$ and find the shape of the set of points which are at the same distance from a given fixed point under the new definition.

2. Two graphs are said to be isomorphic if they have the same number of vertices and whenever two vertices of one are connected by an edge, then there are corresponding vertices connected by an edge and vice versa. A graph is given below. Draw two more graphs which look differently but are isomorphic to the given graph.

Fig. 2

3. Construct a story for the following venn diagram. Copy the diagram and shade $A' \cup B \cap C$;

Fig. 3

4. If $\frac{p+q}{q+r} = \frac{r+s}{s+p}$, find the relation that should govern p, q, r and s .

5. Find the sum to infinity of the series

$$\frac{1}{10} + \frac{2}{102} + \frac{3}{103} + \dots$$

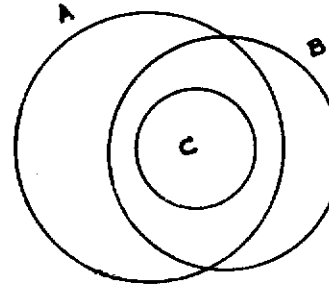


Fig 3

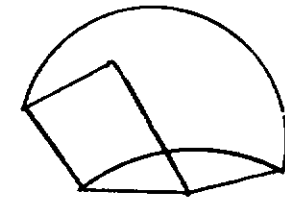


Fig 2

Books for further Guidance:

1. Salkind: *The MAA Problem Book I, II and III* — L.W. Singer Co., USA.
2. Shklarsky et al: *The USSR Olympiad Problem Book*, Freeman, USA.
3. Straszewicz: *Mathematical Problems and Puzzles from the Polish Mathematical Olympiads* — Pergamon.
4. *Hungarian Problem Book I, II*. L.W. Singer, New York, USA.

1. INTRODUCTION

OF all the activities and programmes of a mathematics club, holding a mathematics exhibition, which can be better characterised as an exposition of patterns and powers of mathematical thinking, tops the list; it is at once the most glamorous and popular activity, bringing together the teacher and the pupil in a very purposeful partnership and also the school and the community in a very useful association. Of course, it demands a lot of planned effort in helping pupils prepare exhibits with understanding and to develop readiness to meet outsiders with courage and enthusiasm. It involves careful charting and the pooling of human resources. Pupils skilled in drawing and painting, lettering and border designing, cardboard and fretwork, clay modelling and woodwork, etc., have to be enlisted. Even old boys or former girl students, as the case may be, readily and willingly associate themselves in various capacities with the organisation and running of a mathematics exhibition. The business community is often seen ready to patronise this programme by favouring the organisers with donations or advertisements for the souvenir which becomes a must to make the memorable occasion. In short a mathematics exhibition throws open opportunities for scores of pupils, if not hundreds, to engage themselves in a very colourful and highly rewarding collective endeavour under the guidance of an inspired teacher or band of teachers. Headmasters, headmistresses, principals and managements welcome such a programme, as organising a mathematics exhibition is considered to bring prestige which can be had without much expenditure!

2. ORGANISATION

AS one who has arranged more than forty mathematics expositions, most of them being exclusive ones, I can assert without any fear of contradiction that a mathematics exhibition can be held under all circumstances, provided

the teacher is deeply interested in getting children involved in a creative activity and is keenly aware of its educational value. There was a time when it was thought that there was little to exhibit in School Mathematics and it was even feared that mathematics would suffer by exhibition because of its abstract character. My experience has shown that mathematics, more than any other subject in the school curriculum, needs popularisation through annual expositions, for the very reason that it is abstract. The object of the exposition is naturally to show how mathematics is an abstracting and an abstract science. With the explosion of mathematical knowledge going on in the present day world, creation of a climate for changes in curriculum is one of the inescapable duties and responsibilities of a progressive teacher and this can be discharged through provision of the 'Latest Math' wing in the exhibition. A one-day exhibition is too brief an affair; three-day exhibitions are the normal ones and five-day exhibitions are the ideal ones marked by the release of a souvenir which is at once a memento, a publicity medium and a guidance material for others. Some prefer holding them on holidays but the best thing would be to have the exhibition on working days so that participation of every pupil is assured either in running the exhibition or in seeing it. One of the experiments which I have devised and found workable and beneficial is to secure participation of pupils as expositors of exhibits on a three tier basis—the leaders drawn from the gifted pupils, present and past, the first level volunteers drawn from the bright pupils and the second level volunteers from the average pupils. In case former pupils participate, they will be enrolled as leaders in charge of the various sections in the exhibition, the bright pupils of the top class as deputy leaders and the bright pupils of the next lower class as assistant leaders. They should be put in overall control and supervision of the sections to which they are assigned. They are the persons who would be first trained to be in the know of the exhibits in their sections and they in turn would train up the rest of the expositors. Through a careful time schedule, the level one volunteers can be allowed to move over to every other

exhibit in their section on the second day, while the level two volunteers watch on for getting confidence. On the third day, the level two volunteers take over and stick to their assignments. On the fourth and the fifth days, volunteers are allowed some free time, through substitutes among themselves, to see the whole exhibition. It is essential that two exclusive days should be allotted, one for the pupils of the school, the other for the parents of the school children, to see the exhibition without being mixed with outsiders. Expositors and teachers need badges, other pupils of the school and the parents special tokens or cards for admission to the exhibition, whereas the outsiders would be provided with some tokens or tickets. A nominal charge for admission ticket goes a long way in meeting the printing, stationery and refreshment expenses. But it is not essential. The exhibition should be declared open by at least the head of the school, to mark a formal beginning. It is better if a professor of mathematics is invited to declare open the exhibition under the presidentship of a member of the board of management or a distinguished parent or a prominent leader of the community. Opening can be marked by the cutting of a Mobius strip. The professor of mathematics, if facilitated to have a preview, with the exhibits explained briefly by the leaders of the sections, would be able to present a helpful picture to the audience at the opening function, of the interesting fare awaiting the visitors. The preview may better be thrown open to mass media men also. It is often asked as to how many school rooms would be needed for an exclusive mathematics exhibition. Having gone to the extent of exhibiting about 450 items in 15 rooms involving nearly 250 volunteers, I can say that it all depends on the enthusiasm of the organisers and the participating pupils and the support of the management and the community. It would be attractive if the sections are appropriately named and the exhibition itself is given a caption; Mathematics, the World of Games; or Mathematics, the Adventure of the Mind; or Mathematician, the Ace Model Builder; etc.

Mini math exposition can be arranged at the time of celebrations like school anniversary, founder's day etc.

Enacting episodes from the biographies of mathematicians and from the history of mathematical thought can also form part of the exposition.

Inter school math fairs and inter school math club meets, besides inter school math contests need to be held every year for a get-together of the gifted and interested in mathematics and involvement of parents and public. If the fair is to roll from school to school with relaying in each school for five or six week ends, the fair would attract more participants and visitors.

Preparation of Exhibits: There is a tendency to cover the walls of the exhibition rooms with numerous charts but it is not so useful. Too many charts leave a confused picture in the minds of visitors as they give little scope for the visitors to participate and as they involve the expositors in shouting all the time while reading out or explaining the charts. The resulting din is exhausting to the expositors and dissatisfying to the visitors.

So there should be more of working models than of charts. If need be, each working model may be matched by one or two charts. To prepare working models in school mathematics, it is enough if one could get a supply of bottle-tops, cardboards, strawboards, coloured button papers, marble papers, ruled sheets, square-ruled sheets, graph sheets, drinking straws with pipe cleaning material, coloured twine, thermocole, plywood, clay and plasticine, balsa wood, wire and rods, sticks and plastic sheets. Regarding tools, one should have scissors, fret saw cutting frames, pen-knives, cutting pliers, set of geometrical instruments and sticking paste as the minimum. There should be a good supply of poster colours, writing pens, brushes, erasers, colour pencils and Indian Ink. Letter-stencils would be handy if available for lettering. Use of mirrors, standard measures, physical and spring balances, clocks and stop clocks, electric circuits, etc., makes the exhibition more realistic.

The exhibits should be so prepared that they appeal to the game spirit in man. Visitors should consider it fun to handle each model and 'see' the underlying mathematical principle. There should be scope for experiment and

manipulation. Getting at the mathematical principles through reduction of trial and error processes is highly rewarding in an exhibition.

The exhibits should also be so prepared that they carry an appeal to visitors of all age groups and different backgrounds. The exhibition centre should not be a replica of the class room demanding close attention and serious thought. Exhibits which yield quick results should be given preference.

While each exhibit or chart is being prepared, a tag should be attached to it to show its place in the exhibition. For example, (R 7, 5) will indicate that the item belongs to Room No: 7 and is fifth in the sequence. It presupposes, no doubt, a careful distribution and allocation of exhibits in various sections and drawing up a list, with copies of it kept ready.

4. Exhibits: In the light of my experience, I shall detail the sequence of kinds of exhibits according to the degree of popularity. The popularity of an exhibit is a function of the ease with which the underlying pattern is discovered by an uninitiated visitor. A few samples will be described in each kind and display of these samples can alone constitute a model exposition for math teachers who want guidance in launching an exposition in their schools for the first time.

(i) Building games:

1. Placing rectangular solids representing a^3 (one in number), a^2b and ab^2 (three in each) and b^3 (one in number) and posing the problem of composing them to form a cube (of side $a + b$)

2. Placing 8 wooden cubes with three faces coloured, 12 cubes with two faces coloured, 6 cubes with one face coloured and one cube with no face coloured and posing the problem of composing them to form a cube with colour on all faces. Cubes are all of the same size and the colour used is the same.

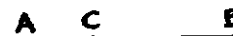
3. Making straw polyhedra with drinking straws and pipe cleaning material.

4. Paper-folded prisms, pyramids, cylinders and cones and regular polygons.

5. Gogo mad or instant insanity game.

(ii) Counting games:

1. Take a straight line segment. Put one point on it. Pose the problem of counting all the segments formed. Put one more point and repeat the problem. And so on.



The number of segments is three ($1 + 2$).

AC, CB, AB.

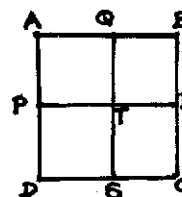


The number of segments is six ($1 + 2 + 3$).

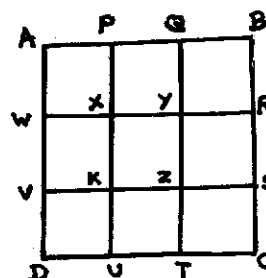
AC, CD, DB, AD, CB, AB.

The sequence of triangular numbers emerges!

2. Take a square 2×2 . Pose the problem of counting the number of big and small squares formed. Next take a square 3×3 and repeat the problem. And so on.



The number of squares is five ($1^2 + 2^2$)
AQIP, QBRT, TRCS, PTSD, ANCD.



The number of squares is fourteen ($1^2 + 2^2 + 3^2$)
APXW, PQYX, ... (nine)
AQZV, WYTD, PBSK, XRCU (four)
ABCD (one)

3. Take a cube $2 \times 2 \times 2$. Pose the problem of counting the number of cubes, big and small, formed. Next take a cube $3 \times 3 \times 3$ and repeat the problem. And so on. In the cube $2 \times 2 \times 2$ one counts nine cubes, big and small ($1^3 + 2^3$) and in the cube $3 \times 3 \times 3$, one counts thirty-six cubes, big and small ($1^3 + 2^3 + 3^3$), and so on.

4. Counting the maximum number of partition in a plane segment with n lines.

5. Counting regions in a circle partitioned by lines, joining, two by two, n points on its circumference.

6. Counting subsets of a set.

(iii) Mathematics through the rules of games:

1. **Vacating game:** Three places (circular regions) are marked on the ground. Three dolls of heights in ascending order are chosen. They occupy one of the places. The game is to get the dolls vacate the place, first occupied, in the least number of steps by following strictly the code of rules given below:

- (i) Dolls should be moved only one at a time;
- (ii) A taller doll should not be moved to a place occupied by a shorter doll. But a shorter doll can be moved to a place occupied by a taller doll;
- (iii) Either of the places unoccupied can be occupied;
- (iv) The places can be used in any order.

By changing the number of dolls to one, two, three and four the formula that the least number of moves is $2^n - 1$, where n is the number of dolls, can be easily discovered.

The game becomes exciting when the rule (iii) is altered by specifying the place to be finally occupied after vacating and when the rule (iv) is altered by specifying that the places must be used in order and the last place is to be occupied finally after vacating the first.

2. **Exchanging game:** Three places in a row are given on either side of a central or neutral place. Three same size cube sets in two different colours are taken and put in a row in the places on either side of the central place left unoccupied. The game is to exchange in the least number of moves one coloured set with the other coloured set by following the code of rules given below:

- (i) Only one cube should be moved at a time;
- (ii) A cube can be put in the next place or in the place got by skipping only one place.

Change the number of cubes in either set to 1, 2, 3, 4 and so on.

The least number of moves is seen to be $n(n + 2)$, where n is the number of cubes on either side of the central neutral place.

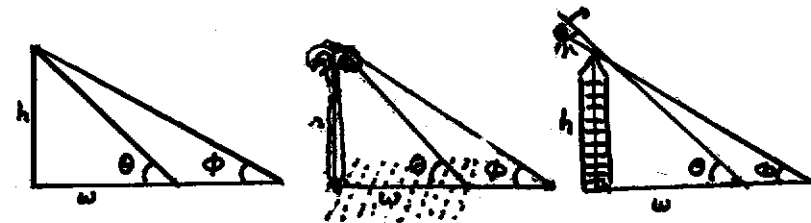
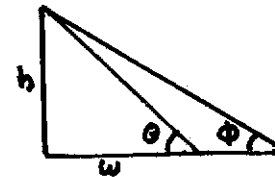
3. Finding the number of elements in a set to satisfy given rules (Finite geometry).

(iv) **Mathematical Models:** How a mathematician sim-

plifies a real problem to its essentials by building a model and interpreting its properties is quite important and must invariably find a place in every mathematics exhibition. For example:

1. Consider the following model, h can be expressed in terms of θ , ϕ and w .

Some of the situations for which this is a model are pictured below:



2. **Building story problems** on the spot for closed and open sentences in mathematics. Examples:

- (i) $5 \times 8 + 1 = 41$
- (ii) $5 \times -3 = 37$

3. Survey problems:

- (i) drawing a straight line across a building;
- (ii) measuring the width of a pond.

4. Beehive and tessellation.

5. Spider's web and similar polygons.

(v) Fallacies and paradoxes:

1. Showing every angle is a right angle.
2. Showing every triangle is isosceles.
3. Showing $1 = 2$.
4. Showing $-1 = +1$.

(vi) Wonders of numberland:

1. Figurate numbers like triangular numbers, square numbers, pentagonal numbers, pyramidal numbers and their properties. These can be beautifully displayed through appropriate arrangement of coloured buttons and bottle-tops in stages suggesting the patterns.

2. Rectangular numbers and composite and prime numbers.

3. Prime numbers classified into those which can be and cannot be expressed as a sum of two squares.

4. Pythagorean triplets.

5. 'Think of a number or numbers' type of games.

6. Magic squares, magic triangles, magic circles and magic hexagons.

7. Divisibility properties.

8. Shortcuts in multiplication and division.

9. Number patterns—'suggest the next three items' type.

10. Fibonacci numbers and their applications.

11. Binary system and its use in games of Nim, ordering a deck of cards, Russian peasant's multiplication, telling the number through window cards arrangement, code, etc.

12. Repeating decimals and their sum (through pattern) and sum to infinity in G.P. Application in some interesting and realistic problems:

(i) Every chocolate bought carries a coupon and ten such coupons get one free chocolate. What is the value of one coupon in terms of a chocolate?

(ii) A master returning home was seen by his dog when he was four kilometres away. The master was walking at 3 km per hour. The dog could run at 6 km per hour. It ran to the master. As soon as it met the master, it ran back to the gate, again saw the master and ran to him. Thus he was running to and fro the gate till the master reached the gate. What is the total distance covered by the dog in its to and fro movements?

13. Digit sum patterns.

14. Designs through number multiples.

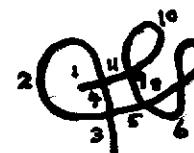
(vii) Topological wonders:

1. Euler's property of polyhedra. Take any prism to start with, as a prism is also a polyhedron. Count its vertices (V), edges (E), and faces (F). One can easily discover that $V + F - E = 2$. It becomes arresting when a polyhedron is made of non-sticky clay or plasticine and plane sections of it are made in various ways and the preservation of the property is looked up in the resulting polyhedron every time.

2. The analogue of Euler's property in two dimensions. Scribble as you like without lifting your hand once you have started. After you finish, count the vertices (ends if any and intersection points), the arcs (the portions between two vertices including loops) and the regions (enclosed ones as well as the outer one). Again one can discover that $V + R - A = 2$.



$V = 7$



$A = 11$



$R = 6$

3. Study and classification of networks.

4. Koenigsberg bridges and traversability problem

5. Mobius strip.

6. Topological stunts in knots.

7. Jordan curve property (use of even and odd number) and its use in wound up belt for spotting out the interstice to hold an inserted pencil or stick, often resorted to in road side gambling game.

8. Betti numbers for connectivity.

Books for further Guidance:

1. Hans Rademacher and Otto Toeplitz: *The Enjoyment of Mathematics* — Princeton, U.P.

2. Eugene F. Krauss: *Taxicab Geometry* — Addition — Wesley Publishing Co., USA.

3. Sutton: *Mathematics in Action* — Harper Brothers, New York.
4. Shockley: *Introduction to Number Theory* — Holt, Rinehart & Winston, USA.
5. Beardsley: *1001 uses of the Hundred Square*, Parker Publishing Co., USA.
6. *Mathematical Applicable Series (14 books)* — Heinemann Educational Books, School Council, U.K.

10. MATHEMATICAL DOODLING FOR POP ENTERTAINMENT

THE programme, which is called "mathematical doodling" ('doodle' = 'scrawl or draw absent-mindedly'), consists of activities which start in an apparently aimless way but soon develop into fascinating exercises with interesting questions cropping up for investigation. To begin with, working sheets containing the illustrative starting steps of the doodling items (with blank spaces to continue the work) are distributed and, in each case, instructions are given as to how to continue further. As one proceeds with the very simple instructions, after a few steps, one comes face to face with surprisingly fascinating patterns, the recognition of which together with discovery of the underlying general principle becomes a source of enjoyment and entertainment. Doodling provides suspense and hours of entertainment for all and particularly for those with limited mathematical background. Doodling gives one freedom to shoot out in any direction dictated by one's fancy.

The object of such a programme is to make people realise that mathematics too, like the fine arts, is capable of providing deep aesthetic satisfaction. Apart from its entertainment value, doodling can be utilised to good advantage by teachers to easily spot out the inquisitive minds among their pupils and to kindle their flair for research.

Some specimens of mathematical doodling are given below: The readers are invited to enjoy the fun by following the instructions and discovering the outcome of their work. They can also try to seek answers to the questions that may crop up in their minds at the end of the work.

1. Sequence of numbers through digit count: Take any natural number N , formed by any number of digits, possibly with repetitions, say, for example, 7222. The digits in ascending orders are three 2's and one 7. We write a new number 3217. In general, from N we derive another number N' , consisting of the digits of N arranged in ascending

order, each digit being preceded by the number denoting its multiplicity in N . Perform this operation repeatedly and see what happens.

7222 — 3217 — 11121317 — 51121317

(After a certain stage the number will be found to regenerate itself and no new numbers will emerge thereafter).

2. Sets of differences of four natural numbers: Starting with a sequence (N_1, N_2, N_3, N_4) of any four natural numbers, derive another sequence (N_1', N_2', N_3', N_4') from it, such that $N_1' = N_1 \odot N_2$, $N_2' = N_2 \odot N_3$, $N_3' = N_3 \odot N_4$, $N_4' = N_4 \odot N_1$. Repeat the operation successively.

8, 312, 568, 4128 — 304, 256, 4072, 4120 — ...
(After a certain stage the differences become zero).

3. Sequence of numbers by addition and multiplication of digits: Take any natural number N consisting of any number of digits, and let s_1 denote the sum of its digits, s_2 the sum of the digits of s_1 , and so on, until finally we get a single digit number s , which may be called the 'reduced sum' of the digits of N . Thus if $N = 534$, $s_1 = 12$, and $s_2 = s = 3$. Obtain the products of each digit of N by the reduced sum s of its digits. Thus in the above case, we obtain the numbers 15, 9, 12, whose reduced sums are respectively 6, 9, 3. We now form a new number $N' = 693$ by arranging in order the digits corresponding to the reduced sums of the digits of the products. See what happens on repeating the process successively.

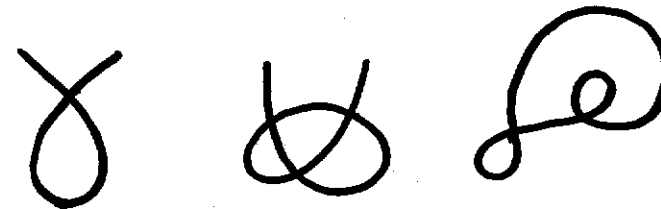
534 — 693 — 999 —

4. Sequences of numbers by sums of squares of digits: Starting with any natural number N , obtain the number N' by taking the sum of the squares of the digits of N . Go on repeating this process and discover the pattern of the sequence. Thus, starting from 47 we have

47 — 65 — 61 — 37 —

5. Pattern in scribbling: It amazes people to discover that even scribbling obeys a definite mathematical law, the only restriction being that it should be done continuously without taking off the pen. In the examples below count the number (p_0) of points, the number (p_1) of arcs

(including loops) and the number (p_2) of regions (inside and outside).



Discover the relationship between p_0 , p_1 , and p_2 .

6. Inside or outside? Everybody will agree immediately that in Fig. 2(i), the point A is inside and the point B outside the (simple) closed curve. Draw rays in different directions from A and B, count the number of points of intersection of each ray with the curve and discover a criterion to decide whether a point is inside or outside a closed simple curve. In Fig. 2(ii) determine just by inspection for each point P, Q and R whether it is inside or outside the closed curve.



Books for further Guidance:

1. Network — *A Mathematics Series* — and Co., (Publishers) Ltd., U.K.

2. Boris A. Kordemsky — *The Moscow Puzzles* — Penguin.

Appendix

A CALENDAR OF BIRTHDAYS OF MATHEMATICIANS

1850	January	15	Sophia Kowlewski (<i>Russian</i>)
1736		25	Lagrange (<i>French</i>)
1810		29	Kummer (<i>German</i>)
1845	March	3	Cantor (<i>German</i>)
1749		23	Laplace (<i>French</i>)
1596		31	Descartes (<i>French</i>)
1815	April	2	Boole (<i>British</i>)
1707		15	Euler (<i>Swiss</i>)
1789		20	Cauchy (<i>French</i>)
1854		29	Poincare (<i>French</i>)
1777		30	Gauss (<i>German</i>)
1746	May	10	Monge (<i>French</i>)
1623	June	19	Pascal (<i>French</i>)
1646	July	1	Leibnitz (<i>German</i>)
1802	August	2	Abel (<i>Norwegian</i>)
1805		3	Hamilton (<i>Irish</i>)
1821		16	Cayley (<i>British</i>)
1602		20	Fermat (<i>French</i>)
1814	September	3	Sylvester (<i>British</i>)
1826		17	Riemann (<i>German</i>)
1831	October	6	Dedekind (<i>German</i>)
1811		25	Galois (<i>French</i>)
1815		31	Weierstrass (<i>German</i>)
1793	November	2	Lobatchewsky (<i>Russian</i>)
1823	December	7	Kronecker (<i>German</i>)
1804		10	Jacobi (<i>German</i>)
1815		10	Augusta Ada Byron Loviace (<i>British</i>)
1887		22	Ramanujan (<i>Indian</i>)
1822		24	Hermite (<i>French</i>)
1623		25	Newton (<i>British</i>)